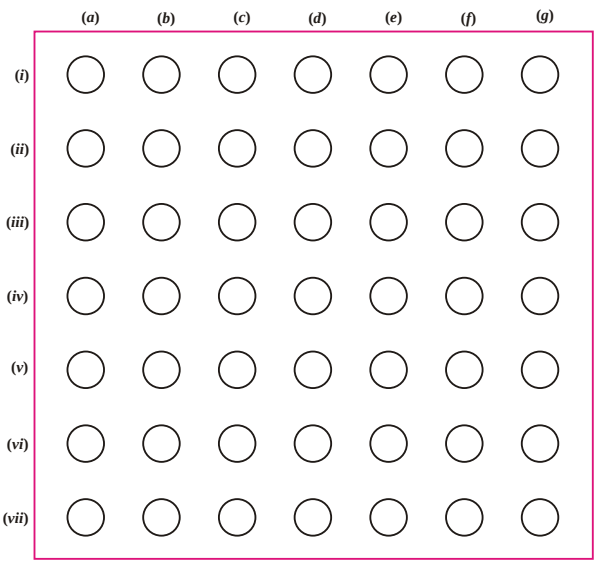
Chapter-One

Real Numbers

Vocabulary Quiz

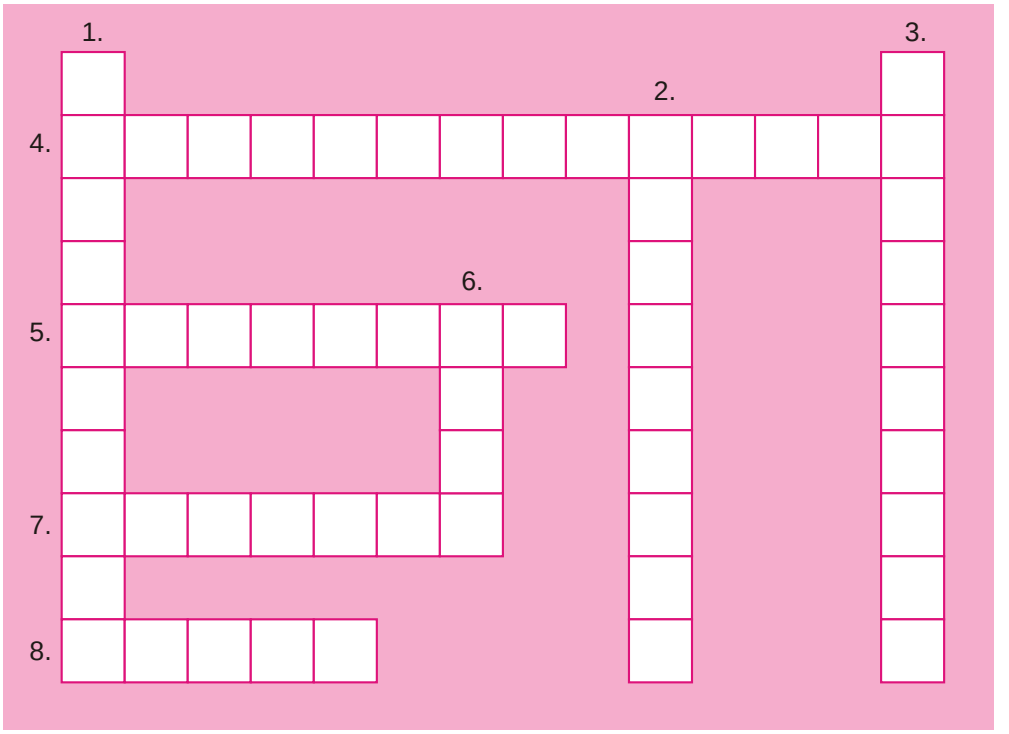
Directions: Match the vocabulary words on the left with the definitions on the right by shading the circles given at the bottom.

(*i*)	Natural numbers	(*a*)	a whole number greater than one that has no factors other than one and itself.
(*ii*)	Whole numbers	(*b*)	the union of rational numbers and irrational numbers.
(*iii*)	Prime number	(*c*)	a number which can be written in the form of $\frac{p}{q}$, where p, q z and q 0
(*iv*)	Integers	(*d*)	the set of natural numbers, zero and negative of natural numbers.
(*v*)	Rational number	(*e*)	the counting numbers 1,2,3, ...
(*vi*)	Irrational number	(*f*)	the counting numbers 1,2,3, ... and zero taken together.
(*vii*)	Real numbers	(*g*)	a number which cannot be written in the form $\frac{p}{q}$, where p,q z and q 0



Puzzle

■ **Look at the Crossword Puzzle given below and fill in the blanks accordingly.**



Across

4. A number whose decimal expansion is terminating or non-terminating recurring.
5. The set of natural numbers, zero and negative of natural numbers.
7. The number system in which all numbers can be written by using only ten digits 0, 1, 2, ...9.
8. $5 \times (3 \times 5 - 3) =$ _____ .

Down

1. The Greek mathematician who was the first to compute digits in the decimal expansion of .
2. Number of integers.
3. Decimal expression of _____ number is non-terminating non-recurring.
6. A number either rational or irrational.

Let us Work Maths

1. Ask your friend to think of a number, call it N.
2. Find (N+1).
3. Now find (N+1) × 2.
4. Subtract 2 from this total.
5. Divide by 2.
6. You will get N.

2. Match the following columns by placing the correct letter in square box.

Column I	Column II
(*i*) 0.150150015000 ...	(*a*) $\frac{1}{9}$
(*ii*) 1.222 ...	(*b*) 9
(*iii*) $729^{\frac{1}{3}}$	(*c*) irrational
(*iv*) $3^{\frac{2}{3}} \quad 3^{\frac{4}{3}}$	(*d*) rational

(*i*) ☐

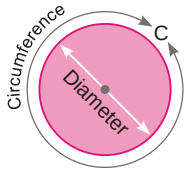
(*ii*) ☐

(*iii*) ☐

(*iv*) ☐

Project Work – 1

$= \frac{\text{Circumference}}{\text{Diameter}}$



Mathematicians : Value of

Archimedes : 3.140845 < <3.14 2557

Aryabhatta : 3.1416

Chung Chi : $\frac{335}{113}$

Direction:

Research the topic indicated to the right and answer the question below based on the topic

Topic: History of

- **Presentation should be based on**
  - Knowledge about in ancient civilisations.
  - Work on by various mathematicians and their estimation for .
  - and circle.
  - List of formulae involving .
  - Some important mathematical problems featuring .

Student should mention all the sources they used to collect the information.

1. Define .	_____ _____ _____

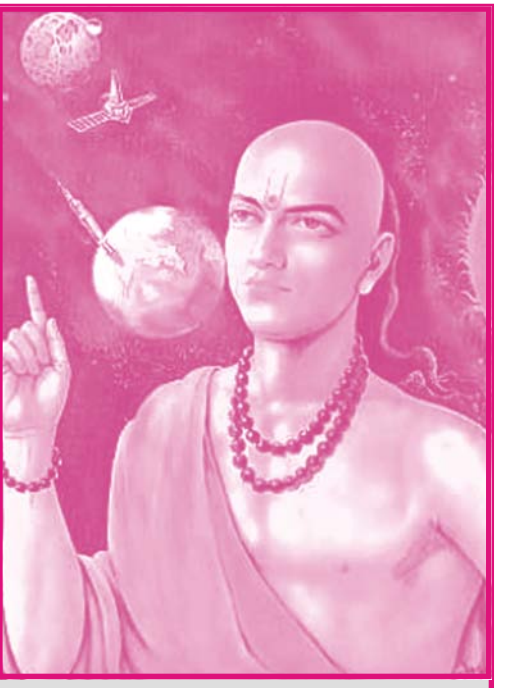
2. Name the mathematicians who proved that is irrational.

3. $\frac{c}{d}$. This seems to contradict the fact that is irrational. How will you resolve this contradiction.

4. What is the approximate value of upto 5 decimal places.

Project -2: Life and Works of Great Indian Mathematician Aryabhatta

Aryabhatta



Aryabhatta, a renowned mathematician and astronomer of ancient India was born in 476 A.D. in Kerala. For his advanced studies, he went to *Kusumapura* where he was the head of an institution. He also set up an observatory at the sun temple in *Tarengana*, Bihar. The famous books written by him are *Aryabhatiya* and *Arya Siddhanta*.

Aryabhatiya was written by him in 499 AD, at the age of 23. He wrote it in verse form including the topics like astronomy, plane trigonometry, spherical trigonometry, arithmetic and algebra. It also includes continued fractions, quadratic equations and a table of sines. This book is divided into four padas or chapters having a total of 121 verses. These padas are Gitikapada (13 verses), Ganitapada (33 verses), Kalakriyapada (25 verses) and Golapada (50 verses). This book presented a number of innovations in mathematics and astronomy, which were influential for many centuries. Arya Siddhanta is a lost work on astronomical computation which is known through the writings of Aryabhata's contemporary Varahamitira and through later mathematicians and commentators like Brahmagupta and Bhaskara.

Contributions

1. Aryabhatta used zero as a placeholder in his place value system.
2. He calculated the value of (Pi) and concluded that it is irrational.
3. He gave the formula for calculating the area of a triangle and a circle.
4. He discussed the concept of sine in his book by the name of Ardha-jya.
5. He worked on the solutions of the equations of the form $ax + b = cy$, known as *diophantine* equations. He called the method of solving such problems as, *Kuttaka* which means breaking into small pieces.
6. He provided elegant results for the summation of series of squares and cubes.
7. He stated and verified pythagoras theorem through examples.
8. Aryabhatta believed that the earth rotates about its own axis.

5. $(x \quad 1)$ is a factor of the polynomial

$(a)\ x^3 \quad x^2 \quad x \quad 1$ $(b)\ x^3 \quad x^2 \quad x \quad 1$ $(c)\ x^4 \quad x^3 \quad x^2 \quad 1$ $(d)\ x^4 \quad 3x^3 \quad 3x^2 \quad x \quad 1$

Sol.

6. Which of the following shows factor of $p \quad 27pq^3$

$(a)\ p(1 \quad 3q)(1 \quad 3q \quad 9q^2)$ $(b)\ p(1 \quad 3q)(1 \quad 3q \quad 9q^2)$ $(c)\ p(1 \quad 3q)(1 \quad 3q \quad 9q^2)$ $(d)\ p(1 \quad 3q)(1 \quad 3q \quad 9q^2)$

Sol.

7. One of the factors of $(25x^2 \quad 1) \quad (1 \quad 5x)^2$ is

$(a)\ (5 \quad x)$ $(b)\ (5 \quad x)$ $(c)\ (5x \quad 1)$ $(d)\ 10x$

Sol.

8. The value of $249^2 \quad 248^2$ is

$(a)\ 1^2$ $(b)\ 477$ $(c)\ 487$ $(d)\ 497$

Sol.

9. If $\frac{x}{y} \quad \frac{y}{x} \quad 1 \ (xy \quad 0)$, the value of $x^3 \quad y^3$ is

$(a)\ 1$ $(b) \quad 1$ $(c)\ 0$ $(d)\ \frac{1}{2}$

Sol.

10. If $49x^2 \quad b \quad 7x \quad \frac{1}{2} \quad 7x \quad \frac{1}{2}$, then the value of b is

$(a)\ 0$ $(b)\ \frac{1}{\sqrt{2}}$ $(c)\ \frac{1}{4}$ $(d)\ \frac{1}{2}$

Sol.

11. Which of the following is a factor of $(x \quad y)^3 \quad (x^3 \quad y^3)$?

(*a*) $x^2 \quad y^2 \quad 2xy$ (*b*) $x^2 \quad y^2 \quad xy$ (*c*) xy^2 (*d*) $3xy$

Sol. _____

_____

_____

12. If $a \quad b \quad c \quad 0$, then $a^3 \quad b^3 \quad c^3$ is equal to

(*a*) 0 (*b*) abc (*c*) $3abc$ (*d*) $2abc$

Sol. _____

_____

_____

Match the Columns

1. Match the following columns by placing the correct letter in square box.

Column I	Column II
(*i*) Degree of the zero polynomial is	(*a*) 3
(*ii*) Zero of zero polynomial is	(*b*) 3
(*iii*) Coefficient of y^2 in the expression $(y \quad 1)^3$ is	(*c*) not defined
(*iv*) If $x^{101} \quad 4$ is divided by $x \quad 1$, the remainder is	(*d*) any real number

(*i*) ☐

(*ii*) ☐

(*iii*) ☐

(*iv*) ☐

2. Match the following columns by placing the correct letter in square box.

Column I	Column II
(*i*) If $x \quad 1$ is a factor of $kx^2 \quad 3x \quad k$, then the value of k is	(*a*) 3
(*ii*) The value of the polynomial $3x^2 \quad 1$, when $x \quad \frac{2}{\sqrt{3}}$ is	(*b*) $\frac{3}{2}$
(*iii*) If $x^3 \quad 3x^2 \quad 3x \quad 1$ is divided by $x \quad 1$, the remainder is	(*c*) 2
(*iv*) $\frac{2^{\frac{4}{3}}}{2^{\frac{1}{3}}}$ is equal to	(*d*) 0

(*i*) ☐

(*ii*) ☐

(*iii*) ☐

(*iv*) ☐

3. Match the following columns for the factors of the given expression by placing the correct letter in square box.

Column I	Column II
(*i*) $x^2 \quad \frac{y^2}{100}$	(*a*) $(a \quad \sqrt{2}b)(a^2 \quad \sqrt{2}ab \quad 2b^2)$
(*ii*) $a^3 \quad 2\sqrt{2}b^3$	(*b*) $(3x \quad y \quad z)(9x^2 \quad y^2 \quad z^2 \quad 3xy \quad yz \quad 3xz)$
(*iii*) $a^3 \quad 2\sqrt{2}b^3$	(*c*) $(a \quad \sqrt{2}b)(a^2 \quad \sqrt{2}ab \quad 2b^2)$
(*iv*) $27x^3 \quad y^3 \quad z^3 \quad 9xyz$	(*d*) $x \quad \frac{y}{10} \quad x \quad \frac{y}{10}$

(*i*) ☐

(*ii*) ☐

(*iii*) ☐

(*iv*) ☐

Project Work - 1

Remainder Theorem

Dividend = (Divisor × Quotient) + Remainder

$p(x) \quad (x \quad a)q(x) \quad r(x)$,

where degree of $p(x)$ > degree of $q(x)$.

Since the degree of $(x \quad a)$ is 1 and the degree of $r(x)$ is less than the degree of $x \quad a$, the degree of $r(x) \quad 0$, i.e., $r(x)$ is a constant say r.

Therefore, $p(x) \quad (x \quad a)q(x) \quad r$

if $x \quad a$, $p(a) \quad (a \quad a)q(a) \quad r \quad r$

Directions:

Research the topic indicated to the right, and answer the questions that follow:

Topic: Remainder Theorem

Representation

- Statement of the theorem.
- Proof of the theorem.
- Application of the theorem.

1. Find the remainder when $x^3 \quad 3x^2 \quad 3x \quad 1$ is divided by $x \quad \frac{1}{2}$.

2. Check whether the polynomial $p(y) \quad 4y^3 \quad 4y^2 \quad y \quad 1$ is a multiple of $2y \quad 1$.

3. Find the remainder when $x^3 \quad ax^2 \quad 6x \quad a$ is divided by $x \quad a$.

Project Work - 2

Factor Theorem

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be a real number such that $p(a)$ 0, then $(x$ $a)$ is a factor of $p(x)$. Conversely, if $(x$ $a)$ is a factor of $p(x)$, then $p(a)$ 0:

By remainder theorem, $p(x)$ when divided by $(x$ $a)$ gives remainder equal to $p(a)$.

$p(x)$ $(x$ $a)q(x)$ $p(a)$

$p(x)$ $(x$ $a)q(x)$ $(\because p(a)$ $0)$

$(x$ $a)$ is a factor of $p(x)$

Conversely, $(x$ $a)$ is a factor of $p(x)$.

$p(x)$, when divided by $(x-a)$ gives remainder zero.

$p(a)$ 0

Direction:

Research the topic indicated to the right, and answer the questions below:

Topic: Factor Theorem

Representation

- Statement of the theorem.
- Proof of the theorem.
- Application of the theorem.

1. Find the value of k, if x 1 is a factor of $2x^2$ kx $\sqrt{2}$.

2. Using factor theorem, factorise x^3 $2x^2$ x 2.

Factorisation of Polynomials

Factorise the following using appropriate identities:

1. $x^2 \quad \frac{y^2}{100}$

2. $4y^2 \quad 4y \quad 1$

3. $2x^2 \quad y^2 \quad 8z^2 \quad 2\sqrt{2}xy \quad 4\sqrt{2}yz \quad 8xz$

4. $27p^3 \quad \frac{1}{216} \quad \frac{9}{2}p^2 \quad \frac{1}{4}p$

5. $64m^3 \quad 343n^3$

6. $27x^3 \quad y^3 \quad z^3 \quad 9xyz$

Project Work - 3 : Life and Works of Mahaviracharya

Mahaviracharya

Mahavira was the great Jaina Mathematician of 10th century who wrote "Ganita-Sara-Samgraha" in 850 AD during the reign of the great Rashtrakuta King Amoghavarsha. This book is divided into nine chapters. He separated astrology from Mathematics. His books inspired many mathematicians in Southern India.

Contributions to Mathematics

1. Instead of the operations of addition and subtraction, he included squares and cubes, square roots and cube roots, summation of arithmetic and geometric series, etc. These operations were based on decimal place-value numeration.
2. He mentioned 24 notational places in his work.
3. He asserted that the square root of a negative number did not exist.
4. He gave the sum of a series whose terms are squares of an A.P. The process of summation of a series from which the first few terms are omitted is called by him Vyutkalita.
5. He was the first Indian mathematician to introduce the method of lowest common multiple which he called Niruddha.
6. He discussed techniques for solving linear, quadratic and higher order equations.
7. He gave empirical rules for area and perimeter of an ellipse.
8. He was the first to give terminology about equilateral, isosceles triangle, rhombus, circle and semi-circle.
9. He gave characteristics of cyclic quadrilateral like that of Bramhagupta. He also gave formula about sides and diagonal of cyclic quadrilateral like the area of a quadrilateral is square root of $[(s\ \ a)(s\ \ b)(s\ \ c)(s\ \ d)]$. If sides of cyclic quadrilateral are a, b, c, d and its diagonals are x and y and

   $x\ \ \sqrt{\frac{ad\ \ bc}{ab\ \ cd}(ac\ \ bd)}$ and $y\ \ \sqrt{\frac{ab\ \ cd}{ad\ \ bc}(ac\ \ bd)}$

   Then $xy\ \ ac\ \ bd$.
10. For the volume of a sphere, he gave an approximate rule as $\frac{9}{2}\ \ \frac{1}{2} * d\ \ * *3$ and an accurate one as 9/10 * 9/2 * (1/2 * d) ** 3 which makes Pi equal to 3.0375.

Direction:

Research the topic indicated to the right and answer the question below based on the topic.

The above biography of Mahaviracharya is given as an reference. There may be many more interesting facts about his life and work. See different sources like books, internet, etc and find the correlation with the information given above.

Topic: Mahaviracharya

- **Highlights**
  - Birth
  - Profession
  - Works
  - Contributions
  - Your findings

  Student should mention all the sources they used to collect the information.

1. To which century did Mahaviracharya belong?

_____

2. Name the famous work of Mahaviracharya. In how many chapters is this book divided?

3. Which operations were included by Mahaviracharya instead of the basic operations of addition and subtraction?

4. What did he call Vyatkalita?

5. What name was given to the lowest common multiple by Mahaviracharya?

6. What approximate value of volume of a sphere was given by Mahaviracharya?

Laboratory Activity

Algebraic Identity $(a \quad b)^3$

Aim

Verification of the identity

$(a \quad b)^3 \quad a^3 \quad 3ab(a \quad b) \quad b^3$ or $(a \quad b)^3 \quad a^3 \quad 3a^2b \quad 3ab^2 \quad b^3$ by using a working model.

Before Start You Must Know

- To make cuboidal and cubic shapes of different dimensions by cutting thermocol sheets.
- That the area of a rectangle is the product of its length and breadth.
- That the volume of a cuboid is the product of its length, breadth and height.

Materials Required

Thermocol sheets of thickness 2 cm, ruler, cutter, adhesive paste/fevicol, water colours and brush.

Theory

We have $(a \quad b)^2 \quad a^2 \quad 2ab \quad b^2$

Multiply both sides by $(a \quad b)$, we get

$$(a \quad b)(a \quad b)^2 \quad (a \quad b)(a^2 \quad 2ab \quad b^2)$$

$$(a \quad b)^3 \quad a(a^2 \quad 2ab \quad b^2) \quad b(a^2 \quad 2ab \quad b^2)$$

$$a^3 \quad 2a^2b \quad ab^2 \quad a^2b \quad 2ab^2 \quad b^3 \quad a^3 \quad 3a^2b \quad 3ab^2 \quad b^3$$

How To Do

For this working model, you will need cuboids of different dimensions.

- Let us take $a = 4$ cm, $b = 2$ cm.

***Step*-1.** Draw two squares of sides 4 cm each on the thermocol sheet and cut out these pieces by using a cutter.

Step-2. Apply adhesive paste on one of the square face (4 cm × 4 cm) and join the two pieces [Fig. (*i*) to (*iii*)].

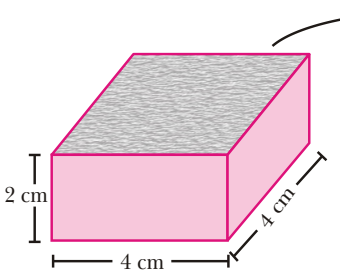


Fig. (i)

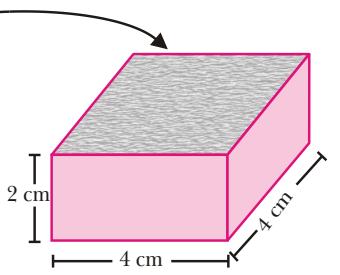


Fig. (ii)

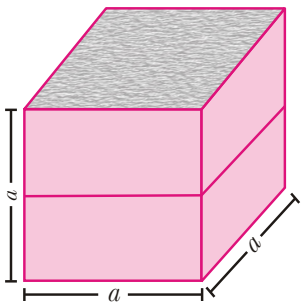


Fig. (iii)

Step-3. You will get a cube of side 4 cm each [Fig (*iii*)]. Paint the cube. Call this cube a^3, because its volume is 4 cm 4 cm 4 cm i.e. a a a units.

Step-4. Draw three squares of sides 4 cm each on the thermocol sheet. Cut out these pieces. You will get three identical cuboidal pieces of dimensions 4 cm 4 cm 2 cm
i.e. a a b. Volume of each of the cuboid is a^2b. See Fig. (*iv*) to (*vi*).

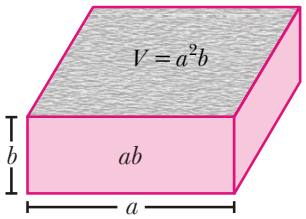


Fig. (iv)

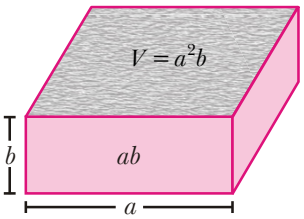


Fig. (v)

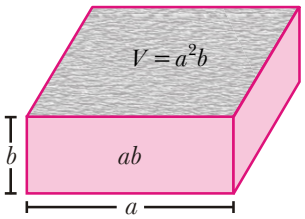


Fig. (vi)

Paint the a^2 surfaces of these cuboids. Also paint the ab surfaces.

Step-5. Draw three congruent rectangles of dimension 2 cm × 4 cm. Cut out these pieces. You will get three identical cuboidal pieces of dimensions 2 cm × 2 cm × 4 cm *i.e.*, $b \times b \times a$. Fig. (*vii*) to (*ix*).
We may call each of these piece as ab^2 because the volume of each cuboid is ab^2 units.
Paint the b^2 faces of each cuboid and also paint the ab faces of each cuboid.

Step-6. Draw a square of side 2 cm on a thermocol sheet and cut it out. This is a cube of volume 2^3 or b^3 paint this cube with colour. Fig. (*x*).

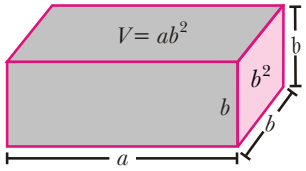


Fig. (vii)

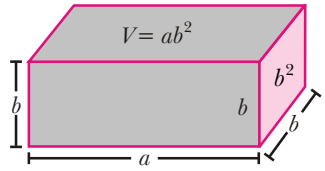


Fig. (viii)

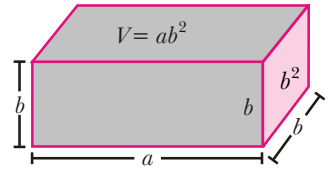


Fig. (ix)

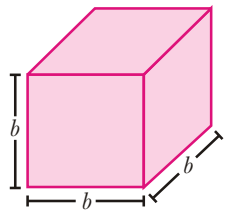


Fig. (x)

We now have two cubes of sides a and b each, three cuboids of sides a, a, b each and three cuboids of sides a, b, b each.

Procedure

Arrange the eight pieces to form a cube so that the side of each face of the new cube is $(a + b)$.

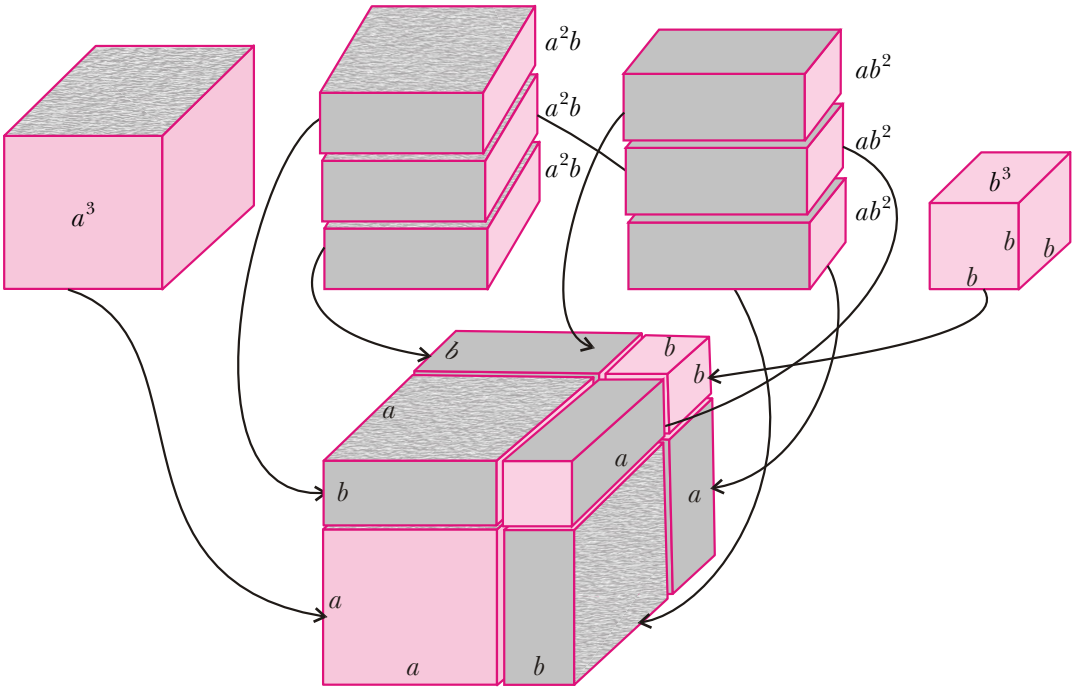


Fig. (xi)

Observations

- Each side of the new cube is $(a \quad b)$.
- The volume of the cube is $(a \quad b)^3$.
- The combined volume of all eight pieces

  which form the new cube $= a^3 \quad (a^2b \quad a^2b \quad a^2b) \quad (ab^2 \quad ab^2 \quad ab^2) \quad b^3$

  or $= a^3 \quad 3a^2b \quad 3ab^2 \quad b^3$

  $(a \quad b)^3 \quad a^3 \quad 3a^2b \quad 3ab^2 \quad b^3$

  Also $(a \quad b)^3 \quad a^3 \quad 3ab(a \quad b) \quad b^3$

Let us Work Maths

This quick technique will enable you to calculate the square of any number that is less than 100 and ends with a 5.

1. To square 75, take the first digit, which is 7, and multiply it by the number obtained on adding 1 to it. In this case, 8.

   $7 \times 8 = 56$

2. Write this number down and put 25 after it to get the correct answer.

   Therefore, $75^2 = 75 \times 75 = 5625$

Rapid Fire Quiz

State which of the following statements are true or false by tick (✓) marking in the box.

		True	False
1.	$x^2 \quad \frac{5x^{\frac{5}{4}}}{\sqrt[4]{x}}$ is a polynomial.	☐	☐
2.	$\sqrt{7}$ is a polynomial of degree zero.	☐	☐
3.	Degree of a zero polynomial is not defined.	☐	☐
4.	Every real number is zero of a zero polynomial.	☐	☐
5.	Polynomial $5t \quad \sqrt{7}$ is a monomial.	☐	☐
6.	Polynomial $x^2 \quad x$ is quadratic.	☐	☐
7.	A linear polynomial has one and only one zero.	☐	☐
8.	A zero of a polynomial, is always zero.	☐	☐
9.	A real number a is root of the equation $p(x)$ if $p(a) \quad 0$.	☐	☐
10.	2 is the remainder when $x^4 \quad x^3 \quad 2x^2 \quad x \quad 1$ is divided by $x \quad 1$.	☐	☐
11.	Zero of the polynomial $p(x) \quad 5x \quad 4$ is 1.25.	☐	☐
12.	The value of $125^2 \quad 124^2$ is 349.	☐	☐
13.	If the value of $(6)^3 \quad (\ 11)^3 \quad (5)^3$ is equal to 1672.	☐	☐
14.	If $x^{151} \quad 151$ is divided by $(x \quad 1)$, the remainder is 150.	☐	☐
15.	If $(x \quad 1)$ is a factor of polynomial $4x^3 \quad 3x^2 \quad 4x \quad k$, then the value of k is 3.	☐	☐
16.	$(x \quad 1)$ is a factor of the polynomial $2x^3 \quad x^2 \quad 2x \quad 1$.	☐	☐
17.	If $a \quad b \quad c \quad 0$, then $a^3 \quad b^3 \quad c^3$ is equal to $3abc$.	☐	☐
18.	The factorisation of $x^3 \quad 6x^2y \quad 12xy^2 \quad 8y^3$ is $(x \quad 2y)(x \quad 2y)(x \quad 2y)$.	☐	☐
19.	The factorisation of $8x^3 \quad 27y^3$ is $(2x \quad 3y)(4x^2 \quad 6xy \quad 9y^2)$.	☐	☐
20.	A linear polynomial can have three terms.	☐	☐
21.	The factors of $x^3 \quad 27$ are $(x \quad 3)(x^2 \quad 3x \quad 9)$.	☐	☐
22.	The coefficient of x^2 in $(x \quad 2)^3$ is $\quad 6$.	☐	☐

Worksheet

1. Write the correct answer for each of the following

(*i*) Which one of the following is a polynomial?

(*a*) $\frac{x \quad 2}{x \quad 2}$ (*b*) $\frac{x^2}{5} \quad \frac{5}{x^2}$ (*c*) $\sqrt{5x} \quad 3$ (*d*) $2x^2 \quad \frac{3x^{5/2}}{x^{3/2}}$

(*ii*) If $f(x) \quad x \quad 5$, then $f(x) \quad f(\ x)$ is equal to

(*a*) 0 (*b*) $2x$ (*c*) 10 (*d*) 10

(*iii*) If $x^{101} \quad 101$ is divided by $x \quad 1$, the remainder is

(*a*) 102 (*b*) 0 (*c*) 1 (*d*) 100

(*iv*) $(x \quad 1)$ is a factor of the polynomial

(*a*) $x^4 \quad x^3 \quad x^2 \quad 1$ (*b*) $x^4 \quad 2x^3 \quad 3x^2 \quad 2$ (*c*) $x^3 \quad x^2 \quad x \quad 1$ (*d*) $x^3 \quad x^2 \quad x \quad 1$

(*v*) The coefficient of x^2 in the expansion of $(x \quad 5)^3$ is

(*a*) 15 (*b*) 75 (*c*) 75 (*d*) 15

2. Write whether the following statements are true or false. Justify your answer.

(*i*) A polynomial can not have more than one zero.

(*ii*) A trinomial can have at most three terms.

3. (*i*) Factorise: $7\sqrt{2}x^2 \quad 10x \quad 4\sqrt{2}$

(*ii*) If $x \quad 2a$ is a factor of $x^5 \quad 4a^2x^3 \quad 2x \quad 2a \quad 3$, find a.

4. (*i*) Without actual division prove that $x^4 \quad 2x^3 \quad 7x^2 \quad 8x \quad 12$ is divisible by $x^2 \quad 5x \quad 6$.

(*ii*) Factorise: $(x^2 \quad y^2)^3 \quad (y^2 \quad z^2)^3 \quad (z^2 \quad x^2)^3$

Performance Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Write the correct answer in each of the following:

(*i*) If $p(x)$ $3x^2$ $6x$ 24, then $p(2)$ is equal to 1

(*a*) 48 (*b*) 24 (*c*) 24 (*d*) 12

(*ii*) The value of 300^2 299^2 is 1

(*a*) 1^2 (*b*) 499 (*c*) 600 (*d*) 599

(*iii*) Rationalising factor of $\frac{1}{7\sqrt{12}}$ is 1

(*a*) $12\sqrt{7}$ (*b*) $12\sqrt{7}$ (*c*) $7\sqrt{12}$ (*d*) $7 2\sqrt{3}$

(*iv*) If x 2 is a factor of the polynomial, $3x^2$ kx 24 then the value of k is 2

(*a*) 6 (*b*) 4 (*c*) 8 (*d*) 6

(*v*) If $\frac{x}{y}$ $\frac{y}{x}$ 1, $(x, y$ 0), the value of x^3 y^3 is 2

(*a*) 1 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) 0

2. State whether the following statements are true or false? Justify your answer.

(*i*) $\sqrt{5}\frac{x^{\frac{5}{2}}}{x^{\frac{1}{2}}}$ 2 is a polynomial.

(*ii*) A binomial may have degree 5. $2 \times 2 = 4$

3. (*i*) Without finding the cubes factorise

$(x$ $2y)^3$ $(2y$ $3z)^3$ $(3z$ $x)^3$

(*ii*) If $(x$ 2) is a factor of $2x^4$ kx^3 $3x^2$ $3x$ 2, find the value of k. $3 \times 2 = 6$

4. (*i*) If both $(x$ 2) and x $\frac{1}{2}$ are factors of px^2 $5x$ r, show that p r,

(*ii*) If x y 12 and xy 27, find the value of x^3 y^3 $4 \times 2 = 8$

Chapter: Three

Introduction to Euclid's Geometry

Vocabulary Quiz

Directions: Match the vocabulary words on the left with the definitions on the right by shading the circles.

(*i*)	Theorems	(*a*)	the assumptions which are obvious universal truth.
(*ii*)	Axioms	(*b*)	statements which are proved, using definition, axioms, previously proved statements and deductive reasoning.
(*iii*)	Ray	(*c*)	part of a line with two end points.
(*iv*)	Line segment	(*d*)	part of a line with one end point.

	(*a*)	(*b*)	(*c*)	(*d*)
(*i*)	○	○	○	○
(*ii*)	○	○	○	○
(*iii*)	○	○	○	○
(*iv*)	○	○	○	○

Let us Work Maths

There are several ways of calculating mentally with the number 11. Here is one way to multiply by 11 and one to see if a number is divisible by 11.

To Multiply a Number by 11

First multiply the number by 10, then add the original number to it.

945×11

$945 \times 10 = 9450$ $\qquad$ $9450+945 = 10395$

To Find Numbers Divisible by 11

- To find out if any number is divisible by 11, start with the digit on the left, subtract the next digit from it, add the next digit, subtract the next, and so on.

  1257795

  $1 - 2 + 5 - 7 + 7 - 9 + 5 = 0$

- If the answer is 0 or 11, then the original number is divisible by 11.

5. What is the condition for consistency of a system of axioms?

6. Why is Axiom 5, in the list of Euclid's axioms, considered as a 'universal truth'?

7. Why do we take a point, a line and a plane as undefined terms in geometry?

Rapid Fire Quiz

State which of the following statements are true or false by tick (✓) marking in the appropriate box.

		True	False
1.	Euclid belongs to Greece.	☐	☐
2.	The side faces of a pyramid are squares.	☐	☐
3.	Pythagoras was a student of Archimedes.	☐	☐
4.	Euclid divided his famous treatise "The element" into 12 chapters.	☐	☐
5.	In ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for public worship.	☐	☐
6.	In ancient India, the shapes of altars used for household rituals were triangles and rectangles.	☐	☐
7.	Euclid's second axiom says that if equals be subtracted from equals, the remainders are equal.	☐	☐
8.	Thales belongs to Egypt.	☐	☐
9.	A solid has two dimensions.	☐	☐
10.	In geometry, we take a point, a line and a plane as defined terms.	☐	☐
11.	Euclid's fourth axiom says that things which coincide with one another are equal to each other.	☐	☐
12.	The number of interwoven isosceles triangles in Sriyantra is eleven.	☐	☐
13.	The Euclidean geometry is valid only for figures in the plane.	☐	☐
14.	In Indus Valley Civilisation the bricks used for construction work were having dimensions in the ratio 4 : 2 : 1.	☐	☐
15.	Thales belongs to the country Babylonia.	☐	☐

Performance Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Write the correct answer for each of the following.

(*i*) Euclid's fifth axiom is (as per order given in the textbook for class IX)

(*a*) If equals be added to the equal, the whole are equals

(*b*) Things which coincide with one another are equal to one another

(*c*) The whole is greater than the part

(*d*) Things which are equal to the same thing are equal to one another. 1

(*ii*) Axioms are assumed

(*a*) theorems (*b*) definitions

(*c*) universal truths specific to geometry (*d*) universal truths in all branches of mathematics 1

(*iii*) It is known that if *a* 2*b* and *c* 2*b* then *a* *c*. The Euclid's axiom that illustrates this statement is

(*a*) second axiom (*b*) fourth axiom (*c*) sixth axiom (*d*) seventh axiom 1

(*iv*) The number of dimensions, a surface has

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3 1

(*v*) Pythagoras was a student of

(*a*) Thales (*b*) Euclid (*c*) Archimedes (*d*) none of these 1

(*vi*) A pyramid is a solid figure, the base of which is

(*a*) only a triangle (*b*) only a square (*c*) only a rectangle (*d*) any polygon 1

(*vii*) Boundaries of surfaces are

(*a*) curves (*b*) lines (*c*) points (*d*) none of these 1

2. Write whether the following statements are true or false. Justify your answer.

(*i*) In geometry, we take a point, a line and a plane as defined terms.

(*ii*) Pyramid is a solid figure, the base of which is a triangle or square or some other polygon and its side faces are equilateral triangles that converges to a point at the top. $2 \times 2 = 4$

3. (*i*) In Fig. 5.1 *AC* *XD*, *C* is the mid-point of *AB* and *D* is the mid-point of *XY*. Using an Euclid's axiom, show that *AB* = *XY*.

(*ii*) It is known that *x* *y* 10 and *x* *z*. Show that *z* *y* 10 $3 \times 2 = 6$

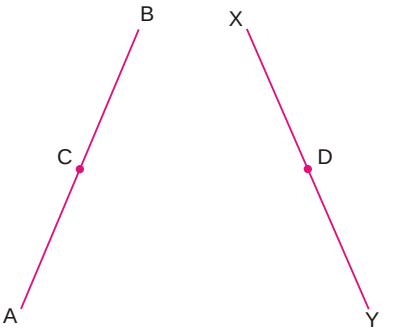


Fig. 5.1

4. (*i*) Prove that two distinct lines cannot have more than one point in common.

(*ii*) Consider two 'postulates' given below.

(*a*) Given two distinct points *A* and *B*, there exist a third point which is in between *A* and *B*.

(*b*) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow Euclid's postulate? Explain. $4 \times 2 = 8$

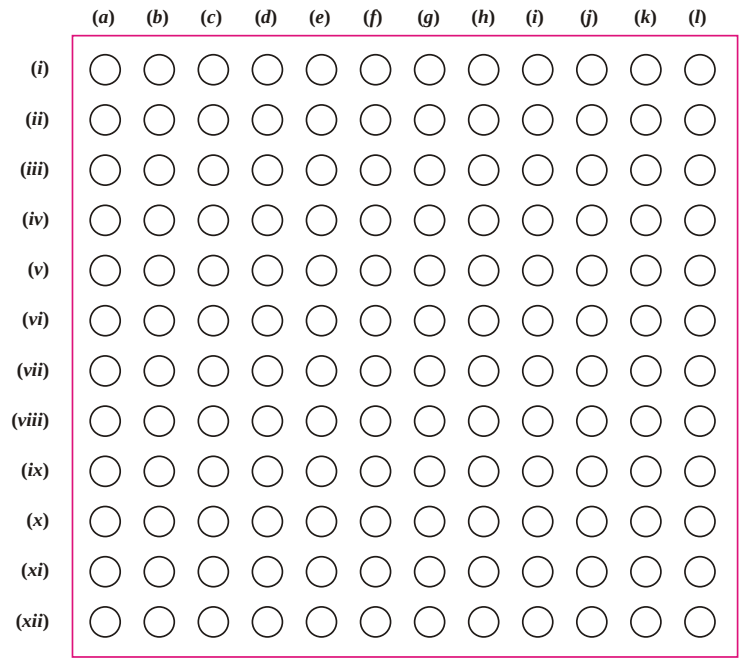
Chapter: Four

Lines and Angles

Vocabulary Quiz

Directions: Match the vocabulary words on the right by shading the circles given at the bottom.

(*i*)	Collinear points	(*a*)	an angle whose measure is between 180° and 360°.
(*ii*)	Concurrent lines	(*b*)	an angle whose measure is more than 90° but less than 180°.
(*iii*)	Acute angle	(*c*)	a triangle in which all the sides are of different measure.
(*iv*)	Right angle	(*d*)	pair of angles sum of whose measure is 180°.
(*v*)	Obtuse angle	(*e*)	a triangle whose two sides are equal.
(*vi*)	Reflex angle	(*f*)	pair of angles sum of whose measure is 90°.
(*vii*)	Complementary angles	(*g*)	an angle whose measure is less than 90°.
(*viii*)	Supplementary angles	(*h*)	a triangle having all sides equal.
(*ix*)	Scalene triangle	(*i*)	points which lie on the same line.
(*x*)	Isosceles triangle	(*j*)	a triangle with one angle 90°.
(*xi*)	Equilateral triangle	(*k*)	three or more lines intersecting at the same point
(*xii*)	Right angled triangle	(*l*)	an angle whose measure is 90°



Activity

To find relation between the corresponding angle formed when a transversal intersect two parallel lines.

- Draw two parallel lines l and m and a transversal t intersecting them. Fig. 4.1.
- Now measure any pair of corresponding angles and find out the relation between them.
- Write your observations and fill in the following table.

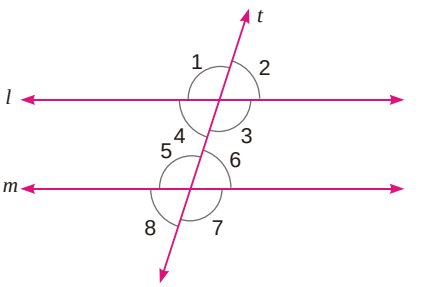


Fig. 4.1

Angle	Angle	Relation
1 =	5 =	
2 =	6 =	
3 =	7 =	
4 =	8 =	

Suggested Activity

1. Draw two non parallel lines and a transversal intersecting them. Find relation between the angles so formed.

Oral Questions

1. What is linear pair axiom?
2. What is a transversal?
3. What is corresponding angle axiom?
4. What is a reflex angle?
5. What are complementary angles?
6. What are supplementary angles?

Assignment

1. In the Fig. 4.2, lines XY and MN intersect at O. If $POY = 90°$ and $a : b = 2 : 3$, find a, b and c.

Sol. _____

_____

_____

_____

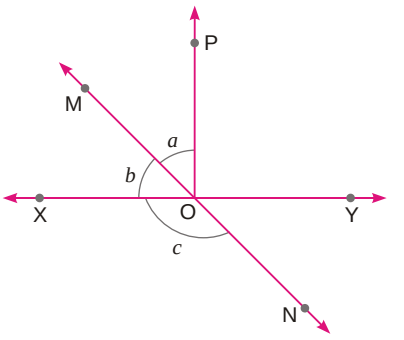


Fig.4.2

2. In Fig. 4.3, if x y w z, then prove that AOB is a line.

Sol. _____

_____

_____

_____

_____

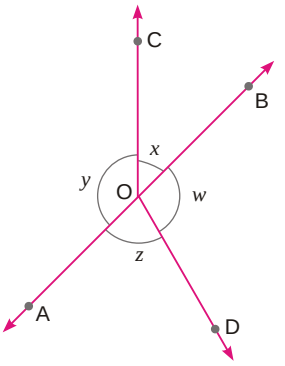


Fig. 4.3

4. (*i*) Prove that sum of the angles of a triangle is 180°.

(*ii*) In ABC (Fig. 4.23), the sides AB and AC of ABC are produced to points E and D respectively. If bisectors BO and CO of CBE and BCD respectively meet at point O, then prove that $BOC \quad 90 - \frac{1}{2} A$.

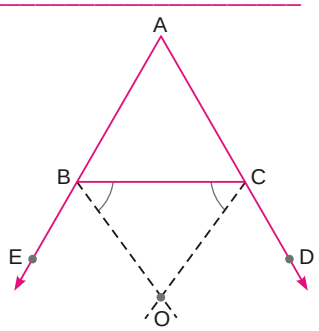


Fig. 4.23

Chapter: Five

Triangles

Word Box

Directions: Complete the statements by writing the word from word box on the spaces provided.

larger	angles	congruent	equilateral
greater	included	isosceles	two
sides	perpendicular bisector		

1. Two figures are _____ , if they are of the same shape and of the same size.
2. Two triangles are congruent if _____ sides and the _____ angle of one triangle are equal to the _____ sides and the included angle of the other triangle.
3. _____ opposite to equal sides of a triangle are equal.
4. _____ opposite to equal angles of a triangle are equal.
5. If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is _____.
6. In a triangle, angles opposite to the longer side is _____ .
7. In a triangle, side opposite to the _____ angle is longer.
8. Sum of any two sides of a triangle is _____ than the third side.
9. A point equidistance from two given points lies on the _____ of the line segment joining the two points.
10. Each angle of an _____ triangle is 60°.

Recreation Time

Count the Triangle

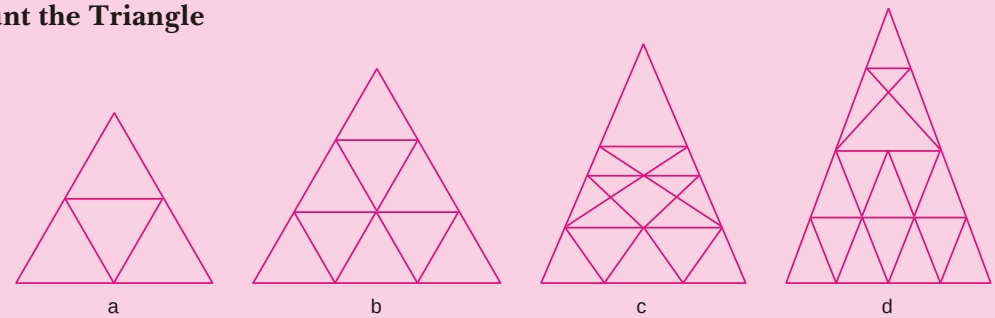
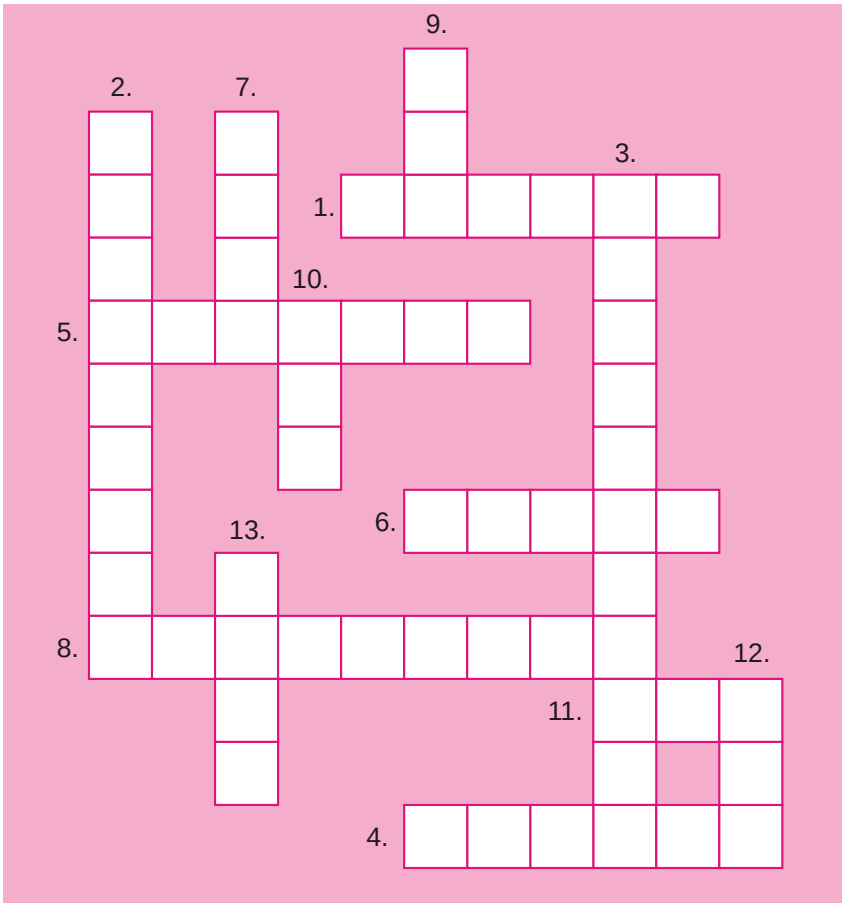


Fig. 5.1

- Can you tell how many triangles are there in each of these figures?

Puzzle

■ **Look at the crossword puzzle given below and fill in the blanks accordingly.**



Across

1. In a triangle, side opposite to the greater angle _____ .
4. _____ opposite to equal sides of a triangle are equal.
5. Angle opposite to greater side of a triangle _____ .
6. Sides opposite to equal angles of a triangle _____ .
8. The sum of any two sides of a triangle is greater than _____ .
11. Congruence criterion when one angle and one side of a right triangle are equal to corresponding site and angle of another right triangle.

Down

2. If two sides and included angle of the triangle are equal to two sides and the included angle of the other triangle, then two triangles are _____.
3. Each angle of an _____ triangle is 60°.
7. In an isosceles triangle altitude from the vertex bisects _____.
9. Number of equal sides of an isosceles triangle.
10. One of the criterions for congruence of triangles.
12. Congruence criterion when three sides of a triangles are equal to the corresponding sides of a another triangle.
13. If all angles of a triangle are equal then all of its _____ are also equal.

2. Is it possible to construct a triangle with lengths of its sides as 8 cm, 6 cm and 15 cm? Give reasons for your answer?

3. In PQR, $PQ = 6$ cm, $QR = 7$ cm and $PR = 9$ cm, which angle of this triangle is largest? Give reason for your answer.

4. In Fig. 5.8, AB and CD are respectively the smallest and longest sides of a quadrilateral $ABCD$. Show that A C and B D.

Sol.

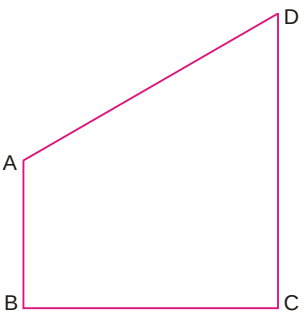


Fig. 5.8

5. In Fig. 5.9, $PR > PQ$ and PS bisects QPR. Prove that $PSR >$ PSQ.

Sol.

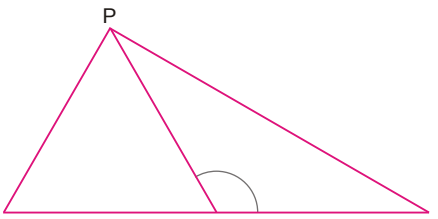


Fig. 5.9

6. Show that in a right triangle, the hypotenuse is the longest side.

Sol.

2. (*i*) Is it possible to construct a triangle with lengths of its sides as 5 cm, 8 cm and 4 cm.

(*ii*) In Fig. 5.20, $PQ = PR$ and Q R. Prove that PQS PRT.

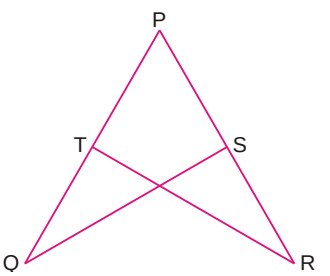


Fig. 5.20

3. (*i*) In Fig. 5.21, D and E are points on side BC of a ABC such that $BD = CE$ and AD AE. Show that ABD ACE.

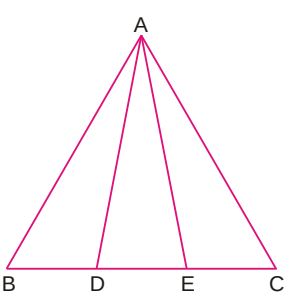


Fig. 5.21

(*ii*) D is any point on side AC of a ABC with $AB = AC$. Show that $CD < BD$.

4. (*i*) ABC is an isosceles triangles in which $AB = AC$. Sides BA is produced to D such that $AD = AB$. Show that BCD is a right angle.

(*ii*) Prove that the perimeter of a triangle is greater than the sum of its three medians.

Performance Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Write the correct answer for each of the following.

(*i*) In triangles ABC and PQR, $AB = AC$, C R and B Q. The two triangles are

(*a*) Isosceles but not congruent (*b*) isosceles and congruent

(*c*) Congruent but not isosceles (*d*) neither congruent nor isosceles 1

(*ii*) In PQR, PQ PR and R 60 . Then Q is equal to

(*a*) 30° (*b*) 120° (*c*) 60° (*d*) 90° 1

(*iii*) In ABC and PQR, $AB = PQ$ and $B =$ Q. The two triangles will be congurent by SAS axiom if

(*a*) $AC = PR$ (*b*) $BC = PQ$ (*c*) $AC = QR$ (*d*) $BC = QR$ 1

(*iv*) Two sides of a triangle are of lengths 7.2 cm and 3.8 cm. The length of the third side of the triangle cannot be

(*a*) 3.5 cm (*b*) 4.5 cm (*c*) 3.2 cm (*d*) 3.8 cm 1

(*v*) In Fig. 5.22, P is

(*a*) 50°

(*b*) 40°

(*c*) 30°

(*d*) 70° 1

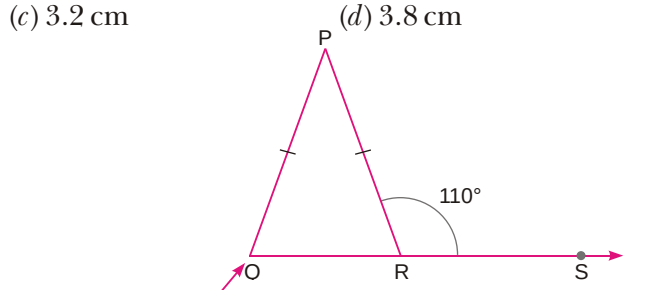


Fig. 5.22

(*vi*) In Fig. 5.23, l is the bisector of A. Then BP is equal to

(*a*) 3.5 cm

(*b*) 5 cm

(*c*) 4 cm

(*d*) 3 cm 2

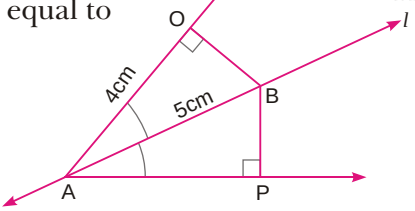


Fig. 5.23

2. (*i*) "If two angles and a side of one triangle are equal to two angles and side of another triangle, then the two triangles must be congruent". Is the statement true? Why?

(*ii*) In Fig. 5.24, $PQ > PR$ and QS and RS are the bisectors of Q and R, respectively. Show that SQ SR $2 \times 2 = 4$

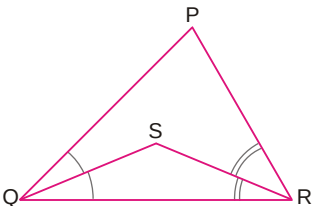


Fig. 5.24

3. (*i*) In Fig. 5.25, ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively. Show that these altitudes are equal.

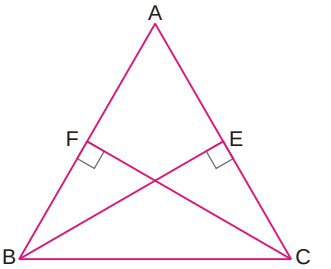


Fig. 5.25

Chapter: Six

Coordinate Geometry

Vocabulary Quiz

Directions: Match the vocabulary words on the left with definitions on the right by shading the circles given at the bottom.

(*i*)	*x*-axis	(*a*)	vertical line in Cartesian plane.
(*ii*)	*y*-axis	(*b*)	horizontal line in Cartesian plane.
(*iii*)	*x*-coordinate of a point	(*c*)	perpendicular distance of the point from the *x*-axis.
(*iv*)	*y*-coordinate of a point	(*d*)	perpendicular distance of a point from the *y*-axis.
(*v*)	Origin	(*e*)	each parts of the plane formed by two axes.
(*vi*)	Quadrants	(*f*)	point of intersection of the axes.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
(*i*)	○	○	○	○	○	○
(*ii*)	○	○	○	○	○	○
(*iii*)	○	○	○	○	○	○
(*iv*)	○	○	○	○	○	○
(*v*)	○	○	○	○	○	○
(*vi*)	○	○	○	○	○	○

Let us Work Maths

This is a quick way to multiply number that differ by 2. It works best if the number in between can easily be squared mentally.

- To multiply 39 by 41, first mentally square the number in between the two: in this case, 40.

  39×41

  $40 \times 40 = 1600$

- When you have squared the number, subtract 1 from the result to find the answer to the calculation.

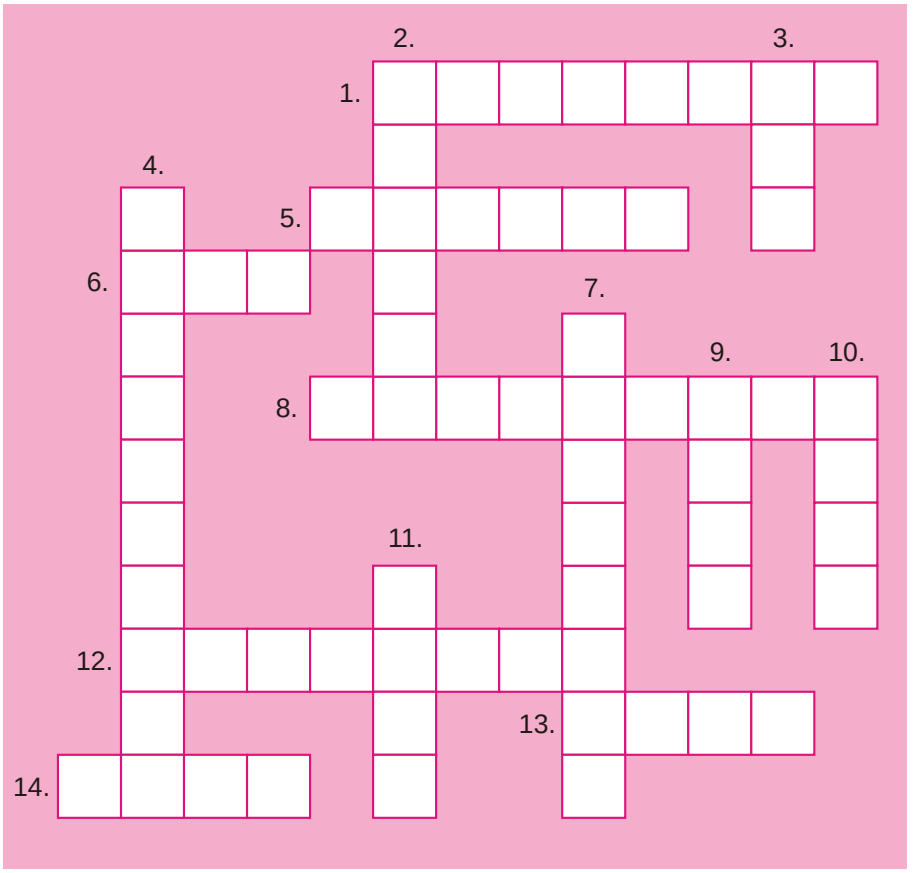
  $1600 - 1 = 1599$

Try to find why this trick work?

Hint: Use the identity $(a \quad b)(a \quad b) \quad a^2 \quad b^2$

Puzzle

■ **Look at the crossword puzzle given below and fill in the blanks accordingly.**



Across

1. The y-coordinate of a point.
5. An equation of the form $ax \quad b \quad 0$, $a \quad 0$, is called _____ equation.
6. Number of origins in a cartesian plane.
8. x-coordinate of any point on x-axis can be _____ .
12. The x-coordinate of a point is called _____ .
11. The ordinate of the point (1, 9) is _____ .
14. The abscissa of a point on y-axis is _____ .

Down

2. The point where x-axis and y-axis meet.
3. Number of coordinate axes.
4. One of the numbers in an ordered pair that represents a point in a cartesian plane.
7. One of the four parts in which the coordinate axes divide the plane.
9. In first quadrant which of the coordinates of a point is positive.
10. The coordinates of a point are always _____ numbers.
11. Any of the coordinate axes is a straight _____ .

A $(-4, 5)$, B $(-3, 4)$, C $(-4, 3)$, D $(-3, 2)$, E $(-5, 2)$, F $(-4, 2)$, G $(-4, 1)$, H $(-2, 1)$, I $(-2, 0)$

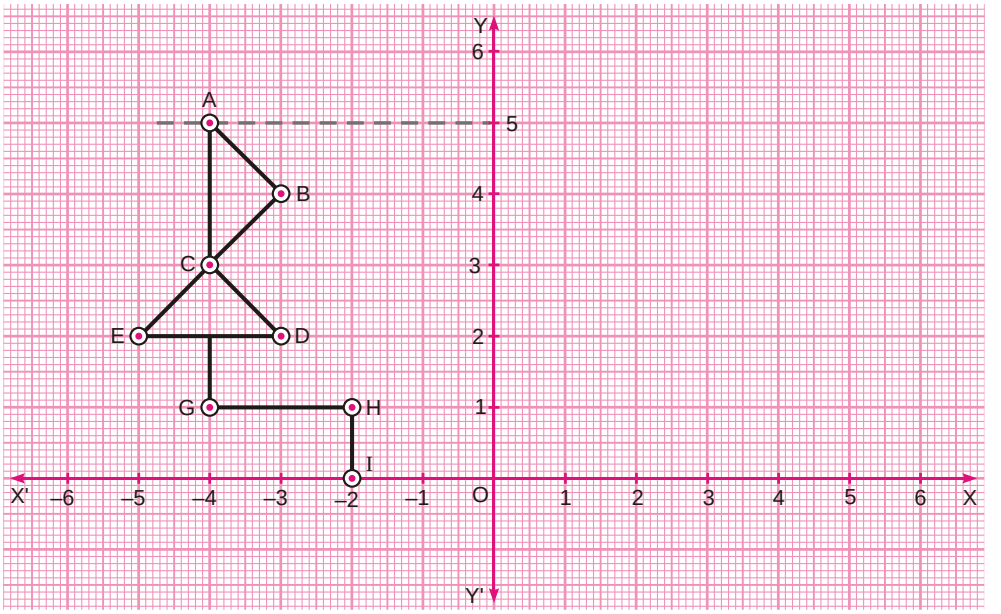


Fig. 6.3

- Draw a line perpendicular to Y-axis through A [Fig. 6.4]. A is at distance of 4 units to the left of Y-axis.
- So locate the point on this perpendicular line at distance of 4 units towards right of Y-axis to find mirror image of point A with respect to Y-axis.

  This point is A' whose coordinates are $(4, 5)$.

  Similarly, the coordinates of the mirror images of the other points w.r.t. to Y-axis are:

  B' $(3, 4)$, C' $(4, 3)$, D' $(3, 2)$, E' $(5, 2)$, F' $(4, 2)$, G' $(4, 1)$, H' $(2, 1)$, I' $(2, 0)$

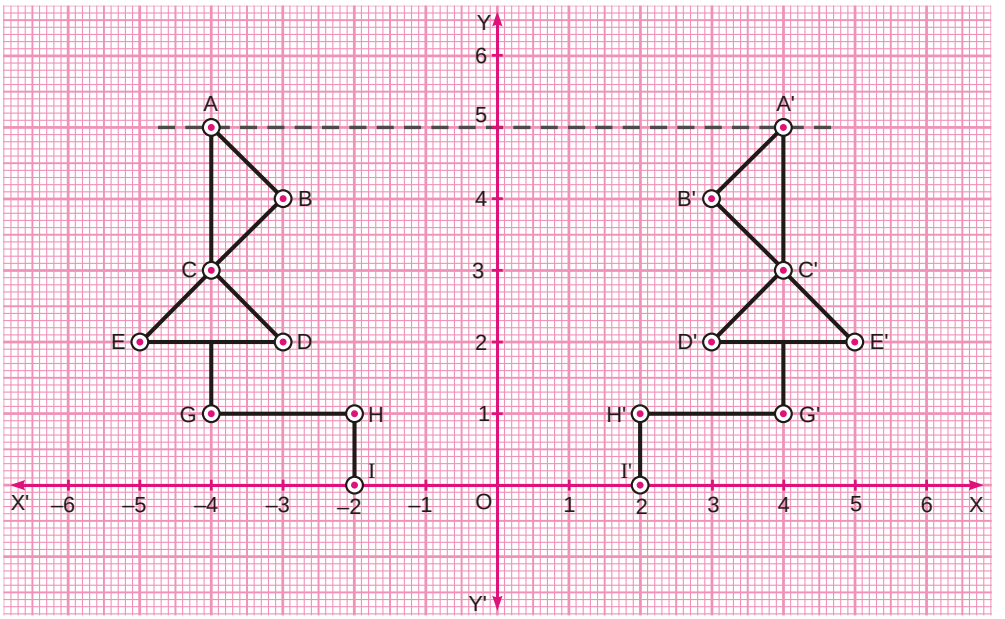


Fig. 6.4

- Join the points to get the reflection of the given figure with respect to Y-axis.

Observations

- The Y-coordinate of all the points remain same after reflection along Y-axis.
- The absolute value of coordinate remains same. Only the sign changes after reflection along Y-axis.

Conclusion

- The given figure and its mirror image along Y-axis form a shape which is symmetric about Y-axis.

Do yourself

- Find the mirror image of the above figure with respect to X-axis. Write your observations and conclusion also.

Rapid Fire Quiz

State whether the following statements are true or false by tick (✓) marking in the square.

		True	False
1.	Point (– 4, –5) lies in the fourth quadrant.	☐	☐
2.	A point both of whose coordinates are negative will lie in third quadrant.	☐	☐
3.	Point (0, 0) lies on x and y axis both.	☐	☐
4.	The perpendicular distance of the point (5, 6) from the x-axis is 5.	☐	☐
5.	Coordinates of a point on the x-axis are of the form (x, 0).	☐	☐
6.	Coordinates of a point on the y-axis are of the form (y, 0).	☐	☐
7.	The coordinates of origin are (0, 0)	☐	☐
8.	Abscissa of point is positive in I and IV quadrants.	☐	☐
9.	The point in which abscissa and ordinate have different signs will lie either in II or in IV quadrant.	☐	☐
10.	If y-coordinate of a point is zero, then this point always lies on y-axis.	☐	☐
11.	Signs of the abscissa and ordinate of a point in the first quadrant are respectively (,).	☐	☐
12.	The point at which the two axes meet is called the origin.	☐	☐
13.	The coordinates of a point are of the form (,) in the second quadrant.	☐	☐
14.	The coordinates of a point which is on y-axis at a distance of 3 units from the x-axis is (3, 0).	☐	☐
15.	The point (5, –2) is same as the point (–2, 5)	☐	☐

(*ii*) Find the area of an equilateral triangle with side x using Hero's formula.

4. (*i*) A park, in the shape of a quadrilateral $ABCD$, has C 90 , AB 9 m, BC 12 m, CD 5 m and AD 8 m. How much area does it occupy?

(*ii*) Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2.

Performance Test

Max. Marks: 25 **Time allowed: 45 minutes**

1. Write the correct answer for each of the following:

(*i*) The area of a right-angled triangle whose legs are 12 cm and 14 cm in length is 1

(*a*) $168\,cm^2$ (*b*) $84\,cm^2$ (*c*) $42\,cm^2$ (*d*) $80\,cm^2$

(*ii*) If the area of an equilateral triangle is $25\sqrt{3}\,cm^2$, then its perimeter is 2

(*a*) 10 cm (*b*) 30 cm (*c*) 15 cm (*d*) 20 cm

(*iii*) The area of an isosceles triangle having base 4 cm and the lengths of one of the equal sides 5 cm is 2

(*a*) $4\sqrt{21}\,cm^2$ (*b*) $84\,cm^2$ (*c*) $42\,cm^2$ (*d*) $2\sqrt{21}\,cm^2$

(*iv*) The area of a triangle with sides 11 cm, 12 cm and 13 cm is 2

(*a*) $6\sqrt{105}\,cm^2$ (*b*) $12\sqrt{105}\,cm^2$ (*c*) $60\sqrt{35}\,cm^2$ (*d*) $6\sqrt{35}\,cm^2$

2. (*i*) Find the area of a triangle, two sides of which are 60 cm, 100 cm and the perimeter is 300 cm.

(*ii*) The area of a regular hexagon of side '*a*' is the sum of the areas of the five equilateral triangles with side '*a*'. Is it true or false? Justify your answer. $2 \times 2 = 4$

3. (*i*) Find the area of an isosceles triangle with base x and one of the equal sides y, using Hero's formula.

(*ii*) Find the area of the trapezium $PQRS$ with height PQ given in the adjoining figure. $3 \times 2 = 6$

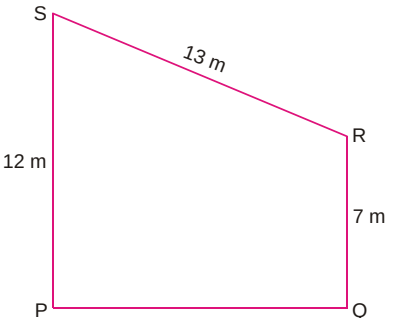


Fig. 7.2

4. (*i*) A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of the grass field will each cow be getting.

(*ii*) A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These, sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs 4 to plough 1 m^2 of the field, find the total cost of ploughing the field. $4 \times 2 = 8$